

The Maximal Amount of Gravitational Waves in the Curvaton Scenario

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The curvaton scenario for the generation of the cosmological curvature perturbation on large scales represents an alternative to the standard slow-roll scenario of inflation in which the observed density perturbations are due to fluctuations of the inflaton field itself. Its basic assumption is that the initial curvature perturbation due to the inflaton field is negligible. This is attained by lowering the energy scale of inflation, thereby highly suppressing the amount of gravitational waves produced during inflation. We compute the power-spectrum of the gravitational waves generated at second order in perturbation theory by the curvaton (isocurvature) perturbations between the end of inflation and the curvaton decay. An interesting property of this contribution to the tensor perturbations is that it is directly proportional to the amount of non-Gaussianity predicted within the curvaton scenario. We show that the spectrum of gravitational waves may be in the range of future gravitational wave detectors.

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Inflation [1, 2] has become the dominant paradigm for understanding the initial conditions for structure formation and for Cosmic Microwave Background (CMB) anisotropy. In the inflationary picture, primordial density and gravity-wave fluctuations are created from quantum fluctuations “redshifted” out of the horizon during an early period of superluminal expansion of the universe, where they are “frozen” [3, 4]. These perturbations at the surface of last scattering are observable as temperature anisotropy in the CMB. The last and most impressive confirmation of the inflationary paradigm has been recently provided by the data of the Wilkinson Microwave Anisotropy Probe (WMAP) mission which has marked the beginning of the precision era of CMB measurements in space [5].

Despite the simplicity of the inflationary paradigm, the mechanism by which cosmological adiabatic perturbations are generated is not yet established. In the standard slow-roll scenario associated to single-field models of inflation, the observed density perturbations are due to fluctuations of the inflaton field itself when it slowly rolls down along its potential. When inflation ends, the inflaton ϕ oscillates about the minimum of its potential $V(\phi)$ and decays, thereby reheating the universe. As a result of the fluctuations each region of the universe goes through the same history but at slightly different times. The final temperature anisotropies are caused by the fact that inflation lasts different amounts of time in different regions of the universe leading to adiabatic perturbations.

An alternative to the standard scenario is represented by the curvaton mechanism [6, 7] where the final curvature perturbations are produced from an initial isocurvature perturbation associated to the quantum fluctuations of a light scalar field (other than the inflaton), the curvaton, whose energy density is negligible during inflation. The curvaton isocurvature perturbations are transformed into adiabatic ones when the curvaton decays into radiation much after the end of inflation.

Contrary to the standard picture, the curvaton mechanism exploits the fact that the total curvature perturbation (on uniform density hypersurfaces) ζ can change on arbitrarily large scales due to a non-adiabatic pressure perturbation which may be present in a multi-fluid system. While the entropy perturbations evolve independently of the curvature perturbation on large scales, the evolution of the large-scale curvature is sourced by entropy perturbations.

During inflation, the curvaton energy density is negligible and isocurvature perturbations with a flat spectrum are produced in the curvaton field σ ,

$$\delta\sigma_{\mathbf{k}} = \left(\frac{H_*}{2\pi} \right), \quad (1)$$

where H_* is the value of the Hubble rate during inflation. After the end of inflation, the curvaton field oscillates during some radiation-dominated era, causing the ratio between its energy density and the radiation energy density to grow, thereby converting the initial isocurvature into curvature perturbation. The energy density ρ_σ will then be proportional to the square of the oscillation amplitude, and will scale like the inverse of the locally-defined comoving volume corresponding to matter domination. On the spatially flat slicing, corresponding to uniform local expansion, its perturbation has a constant value $\delta\rho_\sigma/\rho_\sigma \simeq (\delta\sigma_{\mathbf{k}}/\bar{\sigma}_*)$, where $\bar{\sigma}_*$ is the value of the classical curvaton field during inflation.

The curvature perturbation ζ is supposed to be negligible when the curvaton starts to oscillate, growing during some radiation-dominated era when $\rho_\sigma/\rho \propto a$, where a is the scale factor. After the curvaton decays ζ becomes constant. In the approximation that the curvaton decays instantly it is then given by

$$\zeta_{\mathbf{k}} \simeq r \left(\frac{\delta\sigma_{\mathbf{k}}}{\bar{\sigma}_*} \right), \quad (2)$$

where $r \equiv (\rho_\sigma/\rho)_D$ and the subscript D denotes the epoch of decay. The corresponding spectrum is [6]

$$\mathcal{P}_\zeta^{\frac{1}{2}} \simeq r \left(\frac{H_*}{2\pi\sigma_*} \right). \quad (3)$$

It is nearly scale-invariant under the approximation that the curvaton field is effectively massless during inflation.

The generation of gravity-wave fluctuations is another generic prediction of an accelerated de Sitter expansion of the universe. Gravitational waves, whose possible observation might come from the detection of the B -mode of polarization in the CMB anisotropy [8], may be viewed as ripples of spacetime around the background metric

$$g_{\mu\nu} = a^2(\tau)(d\tau^2 - (\delta_{ij} + h_{ij}) dx^i dx^j), \quad (4)$$

where τ is the conformal time. The tensor h_{ij} is traceless and transverse and has two degrees polarizations, $\lambda = \pm$. Since gravity-wave fluctuations are (nearly) frozen on superhorizon scales, a way of characterizing them is to compute their spectrum on scales larger than the horizon. During a de-Sitter stage characterized by the Hubble rate H_* , the power-spectrum of gravity-wave modes generated during inflation is

$$\mathcal{P}_h(k) = \frac{k^3}{2\pi^2} \sum_{\lambda=\pm} |h_{\mathbf{k}}|^2 = \frac{8}{M_p^2} \left(\frac{H_*}{2\pi} \right)^2, \quad (5)$$

where $M_p = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the Planck scale. Detection of the B -mode of polarization in the CMB anisotropy requires $H_* \gtrsim 10^{12}$ GeV [9].

What about the expected amplitude of gravity-wave fluctuations in the curvaton scenario?

The curvaton scenario liberates the inflaton from the responsibility of generating the cosmological curvature perturbation. Its basic assumption is therefore that the initial curvature perturbation due to the inflaton field is fully negligible. In the standard slow-roll inflationary models where the fluctuations of the inflaton field ϕ are responsible for the curvature perturbations, the power-spectrum of the curvature perturbation is given by

$$\mathcal{P}_\zeta(k) = \frac{1}{2M_p^2 \epsilon} \left(\frac{H_*}{2\pi} \right)^2 \left(\frac{k}{aH_*} \right)^{n_\zeta-1}, \quad (6)$$

where $n_\zeta \simeq 1$ is the spectral index and $\epsilon = (\dot{\phi}^2/2M_p^2 H_*^2)$ is the standard slow-roll parameter. Requiring that the contribution (6) is much smaller than the value required to match the CMB anisotropy imposes $H_* \ll 10^{-5} M_p$. This implies that the curvaton scenario predicts the amplitude of gravitational waves generated during inflation (5) far too small to be detectable by future satellite experiments aimed at observing the B -mode of the CMB polarization (see however [10]).

This is not the full story though. Gravitational waves are inevitably generated at second order in perturbation theory by the curvature perturbations [11, 12, 13]. This scalar-induced contribution can be computed directly from the observed density perturbations and general relativity and is, in this sense, independent of the cosmological model for generating the perturbations. The generation of course takes place after the curvature perturbation is generated.

In the standard scenario, where the curvature perturbation is produced during inflation, the production of tensor modes occurs after inflation when the curvature perturbations re-enter the horizon.

In the curvaton scenario, the production of tensor modes through the curvature perturbations may occur only after the curvaton decays, *i.e.* after the isocurvature perturbations get converted into curvature fluctuations. The energy density of gravitational waves (per logarithmic interval) is given by

$$\Omega_{\text{GW}}(k, \tau) = \frac{k^2}{6\mathcal{H}^2(\tau)} \mathcal{P}_h(k, \tau) \quad (7)$$

and the one generated by the curvature perturbations results to be of order $\Omega_{\text{GW}} \simeq 10^{-20}$, for those modes that re-entered the horizon when the universe was radiation dominated [12, 13].

What we will be concerned about in this paper is the generation of tensor modes by the curvaton perturbations between the end of inflation and the time of curvaton decay. In other words, we are interested in the tensor modes generated at second order when the perturbations are still of the isocurvature nature. This contribution may be larger than the one created by the second-order curvature perturbations after the curvaton decay. In this sense, the spectrum of tensor modes computed in this paper corresponds to the maximal possible amount of gravity waves within the curvaton scenario.

An interesting aspect is that the contribution to the tensor perturbations turns out to be directly proportional to the possibly large amount of Non-Gaussianity (NG) in the CMB anisotropies which is predicted within the curvaton scenario [14, 15]. NG is usually parametrized in terms of the the nonlinear parameter f_{NL} and the latter is predicted to be of the order of $1/r$ in the curvaton scenario; present-day data limit $|f_{\text{NL}}|$ to be smaller than about 10^2 , that is $r \gtrsim 10^{-2}$ [5, 16]. Therefore, the present observational bound on the level of NG in the CMB can already put an upper bound on the amount of tensor modes induced by the curvaton perturbations. This relic gravitational radiation may be particularly relevant in view of the realization that space-based laser interferometers, such as the Big Bang Observer (BBO) and the Deci-hertz Interferometer Gravitational wave Observatory (DECIGO), operating in the frequency range between ~ 0.1 Hz and 1 Hz may achieve the necessary sensitivity [17].

The equation for the second-order gravitational waves

before curvaton decay can be written (neglecting the first-order vector perturbations) as

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} = -4\kappa^2 T_{ij}^{lm} \partial_l \delta\sigma \partial_m \delta\sigma, \quad (8)$$

where $\mathcal{H} = a'/a$ is the Hubble rate, the prime stands for differentiation with respect to the conformal time and $\kappa^2 = 8\pi G_N$. If we define the Fourier transform of the tensor perturbations as follows

$$h_{ij}(\mathbf{x}, \tau) = \sum_{\lambda=\pm} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} h_{\mathbf{k}}^{\lambda}(\tau) e_{ij}^{\lambda}(\mathbf{k}), \quad (9)$$

where the polarization tensors

$$\begin{aligned} e_{ij}^+(\mathbf{k}) &= \frac{1}{\sqrt{2}}(e_i(\mathbf{k})e_j(\mathbf{k}) + \bar{e}_i(\mathbf{k})\bar{e}_j(\mathbf{k})), \\ e_{ij}^-(\mathbf{k}) &= \frac{1}{\sqrt{2}}(e_i(\mathbf{k})\bar{e}_j(\mathbf{k}) - \bar{e}_i(\mathbf{k})e_j(\mathbf{k})) \end{aligned} \quad (10)$$

are expressed in terms of orthonormal basis vectors \mathbf{e} and $\bar{\mathbf{e}}$ orthogonal to \mathbf{k} , the projector tensor in Eq. (8) reads

$$T_{ij}^{lm} = \sum_{\lambda=\pm} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} e_{ij}^{\lambda}(\mathbf{k}) e^{\lambda lm}(\mathbf{k}). \quad (11)$$

In Fourier space, the equation of motion for the gravitational wave amplitude (for each polarization) then becomes

$$\begin{aligned} h_{\mathbf{k}}'' + 2\mathcal{H}h_{\mathbf{k}}' + k^2 h_{\mathbf{k}} &= \mathcal{S}(\mathbf{k}, \tau), \\ \mathcal{S}(\mathbf{k}, \tau) &= 4\kappa^2 \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} e^{+lm}(\mathbf{k}) p_l p_m \delta\sigma_{\mathbf{p}}(\tau) \delta\sigma_{\mathbf{k}-\mathbf{p}}(\tau) \end{aligned}$$

The solution to this equation can be easily found to be

$$h_{\mathbf{k}}(\tau) = \frac{1}{a(\tau)} \int^{\tau} d\tau' g_{\mathbf{k}}(\tau', \tau) a(\tau') \mathcal{S}(\mathbf{k}, \tau'), \quad (13)$$

where $g_{\mathbf{k}}(\tau', \tau)$ is the appropriate Green function either for a radiation- or a matter-dominated period.

We split the perturbations of the curvaton field into a transfer function piece $T_{\sigma}(k, \tau)$ and the primordial fluctuation $\delta\sigma_{\mathbf{k}}$,

$$\delta\sigma_{\mathbf{k}}(\tau) = T_{\sigma}(k, \tau) \delta\sigma_{\mathbf{k}} \quad (14)$$

with the primordial power-spectrum defined by

$$\langle \delta\sigma_{\mathbf{k}} \delta\sigma_{\mathbf{q}} \rangle = \frac{2\pi^2}{k^3} \delta(\mathbf{k} + \mathbf{q}) \mathcal{P}_{\delta\sigma}(k). \quad (15)$$

The power-spectrum of the second-order gravitational waves becomes

$$\begin{aligned} \mathcal{P}_h(k, \tau) &= \\ 16\kappa^4 \int_0^{\infty} dp \int_{-1}^1 d\cos\theta \mathcal{P}_{\delta\sigma}(p) \mathcal{P}_{\delta\sigma}(|\mathbf{k} - \mathbf{p}|) \frac{\sin^4\theta}{a^2(\tau)} \times \\ &\quad \left| \frac{k^3 p^3}{|\mathbf{k} - \mathbf{p}|^3} \left| \int^{\tau} d\tau' a(\tau') g_{\mathbf{k}}(\tau', \tau) T_{\sigma}(p, \tau') T_{\sigma}(|\mathbf{k} - \mathbf{p}|, \tau') \right|^2 \right|, \end{aligned} \quad (16)$$

where $\cos\theta = \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}$. The second-order tensor modes are generated when the various modes k enter the horizon. Meanwhile, the production ends when the curvaton decays. We will assume in the following that the whole generation of tensor modes takes place in the radiation-dominated epoch. This assumption is motivated by requiring that the NG induced by the curvaton is sizeable, which requires the curvaton energy density not to dominate by the time of decay. Since the Hubble rate is given by $\mathcal{H} = 1/\tau$, a given mode k enters the horizon at $\tau_k = 1/k$. Indicating by k_D the mode which enters the horizon at the time of the curvaton decay, $k_D = a(\tau_D)\Gamma$, where Γ is the decay rate of the curvaton, we may write the evolution of the scale factor as

$$a(\tau) = \left(\frac{k_D^2}{\Gamma} \right) \tau. \quad (17)$$

Trading the curvaton decay rate with the temperature at decay T_D , we obtain

$$k_D \simeq 10^{-8} \left(\frac{T_D}{\text{GeV}} \right) \text{ Hz}. \quad (18)$$

After the end of inflation, the zero mode $\bar{\sigma}$ of the curvaton field starts oscillating at $\tau_m \equiv (1/k_D)(\Gamma/m)^{1/2}$, where m is the curvaton mass. Let us first consider those perturbations which enter the horizon when the zero mode $\bar{\sigma}$ of the curvaton decay is already oscillating, that is $k \lesssim (m/\Gamma)^{1/2} k_D$. In this range of wavenumbers, one can show that the curvaton perturbations scale as the zero mode, $\delta\sigma_{\mathbf{k}}(\tau) \sim \bar{\sigma} \sim a^{-3/2}$. This allows to write

$$\begin{aligned} \delta\sigma_{\mathbf{k}}(\tau) &= \left(\frac{\delta\sigma_{\mathbf{k}}}{\bar{\sigma}_*} \right) \left(\frac{1}{k_D \tau} \right)^{3/2} \bar{\sigma}_D \simeq \frac{\zeta_{\mathbf{k}}}{r} \left(\frac{1}{k_D \tau} \right)^{3/2} \bar{\sigma}_D, \\ T_{\sigma}(k, \tau) &= \left(\frac{\bar{\sigma}_D}{\bar{\sigma}_*} \right) \left(\frac{1}{k_D \tau} \right)^{3/2}. \end{aligned} \quad (19)$$

where $\bar{\sigma}_D$ is the value of the curvaton zero mode at the time of decay. If we introduce the variables $x = |\mathbf{k} - \mathbf{p}|/k$ and $y = p/k$ and uses the radiation-dominated Green function $g_{\mathbf{k}}(\tau', \tau) = \sin(k(\tau - \tau'))/k$, it is easy to realize that the main tensor mode production happens at horizon entry, that is at $\tau \simeq \tau_k$. Therefore, the power-spectrum (16) computed at horizon entry is

$$\begin{aligned} \mathcal{P}_h(k, \tau_k) &\simeq 10^2 f_{\text{NL}}^2 \left(\frac{k}{k_D} \right)^6 \left(\frac{\Gamma}{m} \right)^4 \int_0^{\infty} dy \int_{|1-y|}^{1+y} dx \\ &\quad \times \frac{y^2}{x^2} \left(1 - \frac{(1+y^2-x^2)^2}{4y^2} \right)^2 \mathcal{P}_{\zeta}(kx) \mathcal{P}_{\zeta}(ky), \end{aligned} \quad (20)$$

where we have made use of the relations $r = (\kappa^2/3\Gamma^2)(m^2\bar{\sigma}_D^2)$ and $f_{\text{NL}} \sim 1/r$. In the curvaton scenario the resulting curvature perturbation is nearly scale-invariant and we can take $\mathcal{P}_{\zeta} \sim (5 \times 10^{-5})^2$. The remaining integrals in (20) are dominated by the momenta for

which $x \sim y \sim (m/\Gamma)^{1/2}(k_D/k)$. We finally obtain an energy density today of gravitational waves given by

$$\Omega_{\text{GW}} \simeq 10^{-15} \left(\frac{f_{\text{NL}}}{10^2} \right)^2 \left(\frac{k}{k_D} \right)^5 \left(\frac{\Gamma}{m} \right)^{7/2}, \quad (21)$$

valid for $k_D \lesssim k \lesssim (m/\Gamma)^{1/2}k_D$.

For those perturbations which enter the horizon before the zero mode of the curvaton field starts oscillating, that is $k \gtrsim (m/\Gamma)^{1/2}k_D$, the scaling is $\delta\sigma_{\mathbf{k}} \sim a^{-1}$ for $\tau_k \lesssim \tau \lesssim \tau_m^2$, that is till $k \gtrsim ma$. For $k\tau_m^2 \lesssim \tau \lesssim \tau_D$ the scaling is $\delta\sigma_{\mathbf{k}} \sim a^{-3/2}$. Meanwhile, the zero mode $\bar{\sigma}$ remains frozen till the mass of the curvaton becomes larger than the Hubble rate at $\tau \sim \tau_m$. Repeating the steps leading to Eq. (20), we find at horizon entry

$$\begin{aligned} \mathcal{P}_h(k, \tau_k) &\simeq 10^2 f_{\text{NL}}^2 \left(\frac{\Gamma}{m} \right) \int_0^\infty dy \int_{|1-y|}^{1+y} dx \\ &\times \frac{1}{x^4} \left(1 - \frac{(1+y^2-x^2)^2}{4y^2} \right)^2 \mathcal{P}_\zeta(kx)\mathcal{P}_\zeta(ky), \end{aligned} \quad (22)$$

which corresponds to an energy density today of

$$\Omega_{\text{GW}} \simeq 10^{-15} \left(\frac{f_{\text{NL}}}{10^2} \right)^2 \left(\frac{\Gamma}{m} \right), \quad (23)$$

valid for $k \gtrsim (m/\Gamma)^{1/2}k_D$. Finally, let us note that the redshifted gravitational wave at later times is always larger than \mathcal{S}/k^2 . Therefore the power-spectrum of gravity waves produced by the curvaton fluctuations is always bigger than the one generated by the second-order curvature perturbations which is inevitably generated when the cosmological perturbations acquire their adiabatic nature.

There is a simple physical motivation for the fact that the amount of gravity waves generated by the curvaton decay is enhanced by powers of $1/r$ with respect to the

one produced by ordinary second-order curvature perturbations. Indeed, being the final adiabatic perturbations generated by the curvaton isocurvature perturbations, the smaller the amount of curvaton energy density at decay is, the larger the curvaton fluctuations have to be: $\delta\sigma_{\mathbf{k}} \propto \zeta_{\mathbf{k}}/r$, see Eq. (2). The isocurvature perturbations giving rise to gravity waves are therefore parametrically larger at horizon entry than the second-order curvature perturbations.

From our findings we deduce that the amount of gravitational waves in the perturbative regime $\Gamma \lesssim m$ can be as large as $\Omega_{\text{GW}} \simeq 10^{-15}$ maximazing the observationally allowed NG in the CMB anisotropies. This is quite intriguing since such a spectrum is at the range of, *e.g.*, BBO and DECIGO interferometers. To be in the right frequency range, between 10^{-1} and 1 Hz, one has to impose that the curvaton decays at $T_D \lesssim 10^8$ GeV. Furthermore, the correlated BBO interferometer proposal claims a sensitivity down to $\Omega_{\text{GW}} \simeq 10^{-17}$. This would require $\Gamma \gtrsim 10^{-2}m$ in units of $(10^2/f_{\text{NL}})^2$. As a final remark, we point out that the expressions (21) and (23) are also applicable to the so-called modulated reheating scenario in which the curvature perturbations are due to the fluctuations of some light field parametrizing the inflaton decay rate [18]. In such a case though the NG nonlinear parameter f_{NL} does not exceed unity [15].

In summary, we have computed the maximal amount of tensor perturbations which may be generated within the curvaton scenario. It is directly proportional to the square of the nonlinearity parameter f_{NL} which parametrizes the NG in CMB anisotropies. The present observational bound on the level of NG in the CMB data sets already an upper bound on the amount of tensor modes induced by the curvaton perturbations. On the other hand, a possible future detection of a large NG signal in CMB anisotropies would suggest a quantity of gravity waves at range of future gravitational wave detectors.

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